THE SIGNATURE OF GERRYMANDERING IN *RUCHO V. COMMON CAUSE*

Andrew Chin,* Gregory Herschlag,** & Jonathan Mattingly***

I. INTRODUCTION 1242

II. THE EMERGENCE OF THE ENSEMBLE APPROACH 1249
A. The Search for a Justiciable Standard 1249
B. The Focus on Partisan Symmetry 1253
C. The Motivation for the Ensemble Approach 1256

III. THE ENSEMBLE AT TRIAL 1258
A. Procedural History 1258
B. Generating the North Carolina Ensemble 1260
C. Analyzing the North Carolina Ensemble 1265
D. Adjudicating the "Signature of Gerrymandering" 1267

IV. THE ENSEMBLE ON REMAND 1269
A. Gill*'s "Threshold Questions"* 1269
B. District-Specific Standing 1270
C. Justiciability 1274

V. CONCLUSIONS AND FUTURE WORK 1275

---

* Professor of Law, University of North Carolina; J.D., Yale Law School; D.Phil. (Mathematics), University of Oxford. Chin gratefully acknowledges the support and encouragement of Guy-Uriel Charles, Moon Duchin, and the Metric Geometry and Gerrymandering Group, the helpful comments of participants in law faculty workshops at Duke University, Elon University, and the University of North Carolina, the insightful suggestions of Deborah Gerhardt, and the dedicated and diligent efforts of his research assistant Graeme Earle throughout the course of this project.

** Assistant Research Professor of Mathematics, Duke University; Ph.D. (Mathematics), University of North Carolina. See below for acknowlegments.

*** Professor of Mathematics and Professor of Statistical Science, Duke University; Ph.D. (Applied and Computational Mathematics), Princeton University. Herschlag and Mattingly acknowledge collaboration on this topic with a succession of students from 2013–2018 including Christy Vaughn Graves (through Duke PRUV & Data+) and Sachet Bangia, Bridget Dou, Sophie Guo, Han Sung Kang, Justin Luo, Robert Ravier, Samuel Eure, Rahul Ramesh, and Lisa Lebovici (through Duke Data+). They are grateful for the support of the Information Initiative and the Dean's and Provost's offices at Duke University. They also acknowledge useful conversations around these topics with E. Speas, H. Pfister, S. Mukherjee, S. Epstein, C. Mackie, T. Ross, G. Charles, W. Pegden, M. Duchin, B. Esselstyn, and M. Bernstein.

I. INTRODUCTION

In recent years, the United States' mathematical community has been directing unprecedented attention to the problem of partisan gerrymandering,[1] aided by computational advances[2] and spurred by litigation challenging the spate of extreme partisan redistricting that followed the 2010 census.[3] As North Carolina scholars who have been involved in these efforts through the Quantifying Gerrymandering research group in the Department of Mathematics at Duke University,[4] we have developed and witnessed the emergence of promising new statistical methods for identifying partisan gerrymandering and quantifying its effects. One of us also provided expert testimony in *Common Cause v. Rucho*[5] based

---

1. *See, e.g.*, Carrie Arnold, *The Mathematicians Who Want to Save Democracy*, 546 NATURE 200, 200–02 (June 8, 2017); Nate Cohn & Quoctrung Bui, *How the New Math of Gerrymandering Works*, N.Y. TIMES (Oct. 3, 2017), https://www.nytimes.com/interactive/2017/10/03/upshot/how-the-new-math-of-gerrymandering-works-supreme-court.html; Dawn Chan, *A Summer School for Mathematicians Fed Up with Gerrymandering*, NEW YORKER (Aug. 8, 2017), https://www.newyorker.com/tech/annals-of-technology/a-summer-school-for-mathematicians-fed-up-with-gerrymandering; Erica Klarreich, *Gerrymandering is Illegal, but Only Mathematicians Can Prove It*, WIRED (Apr. 16, 2017, 8:00 AM), https://www.wired.com/2017/04/gerrymandering-illegal-mathematicians-can-prove; Stephen Ornes, *Math Tools Send Legislators Back to the Drawing Board*, 115 PROC. NAT'L ACAD. SCI. 6515 (2018).

2. *See, e.g.*, Wendy K. Tam Cho & Yan Y. Liu, *Toward a Talismanic Redistricting Tool: A Computational Method for Identifying Extreme Redistricting Plans*, 15 ELECTION L.J. 351, 357 (2016) (observing the recent "proliferation of significant computing power" and presenting an approach "to integrate technological advances with our articulated strategy for analyzing, contextualizing, and understanding redistricting plans").

3. *See, e.g.*, DAVID DALEY, RATF**KED xv (1st ed. 2016) (describing activities of the Republican State Leadership Committee's Redistricting Majority Project ("REDMAP")); Daniel P. Tokaji, *Gerrymandering and Association*, 59 WM. & MARY L. REV. 2159, 2173 (2018) (describing the post-2010 redistricting cycle as "witness[ing] some of the most extreme partisan gerrymanders ever"); Matt Pancia, *Math Equations Could Help Fight Gerrymandering*, *in* GERRYMANDERING AND VOTING DISTRICTS 25, 25–27 (Rita Santos ed. 2018) (same); Michael Li et al., *The State of Redistricting Litigation*, BRENNANCENTER.ORG (Apr. 25, 2019), https://www.brennancenter.org/blog/state-redistricting-litigation (tracking the status of pending redistricting litigation across the United States).

4. *See* QUANTIFYING GERRYMANDERING, https://sites.duke.edu/quantifyinggerrymandering (last visited Apr. 30, 2019).

5. 318 F. Supp. 3d 777, 870–74 (M.D.N.C. 2018) (discussing testimony of Jonathan Mattingly).

on the group's analysis of North Carolina's congressional map.[6] Another of us filed an amicus brief in *Rucho*'s appeal now pending before the United States Supreme Court.[7] We are writing this Article with the threefold aim of explaining how the analysis was performed, how it was used to substantiate the plaintiffs' claims at trial and on remand, and crucially, how it may serve to address the justiciability concerns that have long attended the Supreme Court's partisan gerrymandering jurisprudence and have represented the legal context for our work.

The justiciability of equal protection challenges to partisan gerrymandering has been hanging in the balance since 2004, when in *Vieth v. Jubelirer*,[8] Pennsylvania voters challenged a 2002 plan under which Republicans were expected to win thirteen of nineteen congressional seats on less than fifty percent of the statewide popular vote.[9] The plaintiffs had proposed a discriminatory effect test whereby a court would determine whether "the 'totality of circumstances' confirms that the map can thwart the plaintiffs' ability to translate a majority of votes into a majority of seats."[10] A majority of the Court found this test irrelevant and unmanageable,[11] and four justices voted to hold equal protection and elections clause challenges to partisan gerrymandering non-justiciable.[12] Justice Anthony Kennedy's pivotal concurrence, however, left open the possibility of "new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties."[13]

Last year, the plaintiffs in *Gill v. Whitford*[14] proposed the efficiency gap as one such new method of analysis in connection with their equal protection challenge to legislative redistricting in Wisconsin.[15] The efficiency gap is a

---

6. *See* Decl. of Jonathan Mattingly, Common Cause v. Rucho, 318 F. Supp. 3d 777 (No. 1:16-CV-1026-WO-JEP), http://s10294.pcdn.co/wp-content/uploads/2016/05/Expert-Report-of-Jonathan-Mattingly.pdf.

7. Br. of 27 Election Law, Scientific Evidence, and Empirical Legal Scholars as Amici Curiae in Support of Appellees, Rucho v. Common Cause, No. 18-422 (S.Ct. filed Mar. 8, 2019) (identifying Andrew Chin as counsel for amici).

8. 541 U.S. 267 (2004).

9. *See* Vieth v. Pennsylvania, 188 F. Supp. 2d 532, 536 (M.D. Pa. 2002).

10. *Vieth*, 541 U.S. at 286–87.

11. *See id.* at 287–90; *id.* at 308 (Kennedy, J., concurring in the judgment) (noting agreement with Part III of the plurality opinion).

12. *See id.* at 305–06 (plurality opinion).

13. *Id.* at 313.

14. 138 S. Ct. 1916, 1924 (2018).

15. *See id.* at 1932–33 (describing efficiency gap) (citing Nicholas O. Stephanopoulos & Eric M. McGhee, *Gerrymandering and the Efficiency Gap*, 82 U. CHI. L. REV. 831 (2015)); Eric McGhee, *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGIS. STUD. Q. 55 (2014).

"single calculation . . . that compares each party's respective 'wasted' votes across all legislative districts," where a wasted vote is one "cast for a losing candidate or for a winning candidate in excess of what that candidate needs to win."[16] As a succinct measure of the statewide disparity between the ability of the major political parties to convert votes into legislative seats, the efficiency gap was designed in large part to address Justice Kennedy's justiciability concerns in *Vieth*.[17] In *Gill*, a three-judge district court panel credited the historically large efficiency gaps observed in Wisconsin's recent elections as evidence that substantial Republican advantages due to partisan considerations in redistricting would likely persist for the life of the plan.[18] As one of us argued as an *amicus curiae* in the case,[19] these findings would have readily survived appellate review under the deferential standard applicable to a trial court's weighing of statistical evidence.[20] *Gill* therefore represented, in our colleague Guy-Uriel Charles's words, "the perfect opportunity for the [Supreme] Court to address the political gerrymandering question once and for all."[21]

On appeal, however, the Supreme Court in *Gill* unanimously held that the equal protection injury to a voter from partisan gerrymandering is district-specific.[22] In vacating and remanding the case for lack of standing,[23] the Court faulted the efficiency gap for measuring only "the effect that a gerrymander has on the fortune of political parties" and not "the effect that a gerrymander has on the votes of particular citizens."[24] In addition, the Court found the efficiency gap incapable of distinguishing between districts with naturally occurring and deliberately manipulated concentrations of Democratic voters.[25] Beyond these objections, mathematicians and social scientists have identified other anomalies in the efficiency gap formula that the Court left unaddressed.[26]

---

16. *Gill*, 138 S. Ct. at 1924 (citation omitted).

17. *See* Stephanopoulos & McGhee, *supra* note 15, at 895–99.

18. *See* Whitford v. Gill, 218 F. Supp. 3d 837, 905–06 (W.D. Wis. 2016).

19. *See* Br. of 44 Election Law, Scientific Evid., and Empirical Legal Scholars as *Amici Curiae* in Supp. of Appellees, Gill v. Whitford, 138 S. Ct. 1916 (2018) (No. 16-1161) [hereinafter *Gill* Amicus Brief].

20. *See id.* at 18–19.

21. Guy-Uriel Charles & Luis E. Fuentes-Rohwer, *Judicial Intervention as Judicial Restraint*, 132 HARV. L. REV. 236, 237 (2018).

22. Gill v. Whitford, 138 S. Ct. 1916, 1930 (2018).

23. *See id.* at 1934.

24. *Id.* at 1933.

25. *See id.*

26. *See* Mira Bernstein & Moon Duchin, *A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap*, 64 NOTICES AM. MATHEMATICAL SOC'Y 1020, 1022–24 (2017) (explaining, *inter alia*, that the efficiency gap enshrines the judicially

Justice Kennedy has since retired,[27] but plaintiffs around the country are continuing to respond to his call for a justiciable approach to quantifying the harms of partisan gerrymandering. Most promisingly, advances in computationally randomized simulations of the redistricting process have enabled plaintiffs to produce large sets (ensembles) of compliant plans from which courts can statistically infer discriminatory intent and effect. These efforts have already successfully supported a state constitutional claim, with dramatic results. In Pennsylvania, where the League of Women Voters challenged the state's congressional plan under the Pennsylvania constitution's equal protection guarantee,[28] expert witness Jowei Chen generated two sets of 500 plans to illustrate "the potential range of redistricting plans attempting to apply the traditional redistricting criteria."[29] The Pennsylvania Supreme Court found Chen's plans to be compelling evidence that the enacted plan unconstitutionally "subordinate[d] the traditional redistricting criteria in the service of partisan advantage."[30] Another expert witness, Wesley Pegden, presented a more extensive and rigorous ensemble analysis involving approximately one trillion randomly generated variations.[31] Pegden's approach is especially promising as it is potentially capable of generating ensembles from which statistically significant findings can be inferred.[32] The court found that Pegden's methodology withstood a challenge from defendants' expert Wendy Cho, and supported the conclusion that the challenged map was "a statistical outlier as compared to maps with nearly identical population equality, contiguity, compactness, and number of county splits."[33] The court ordered a

---

unrecognized principle that "the seat lean should be twice the vote lean" and incentivizes the drawing of districts with a 75–25 partisan split, but concluding that the Supreme Court should accept the efficiency gap "as a starting point in building a test to show when entrenched partisan advantage has risen to the level of vote dilution of political opponents"); Christopher P. Chambers et al., *Flaws in the Efficiency Gap*, 33 J. L. & POL. 1, 33 (2017) (making similar criticisms, and concluding that "we must work harder to find" judicially manageable standards for partisan gerrymandering); *see also* Whitford v. Gill, 218 F. Supp. 3d 837, 949–50, 958–59, 965 (W.D. Wis. 2016) (Griesbach, J., dissenting) (making similar criticisms, and concluding that "the efficiency gap theory . . . fatally relies on premises the courts have already rejected").

27. *See* Michael D. Shear, *Supreme Court Justice Anthony Kennedy Will Retire*, N.Y. TIMES (June 27, 2018), https://www.nytimes.com/2018/06/27/us/politics/anthony-kennedy-retire-supreme-court.html.

28. League of Women Voters v. Pennsylvania, 178 A.3d 737, 741 (Pa. 2018).

29. *See id.* at 818 (discussing expert testimony of Dr. Chen).

30. *See id.*

31. *See id.* at 776–77.

32. *See generally* Maria Chikina et al., *Assessing Significance in a Markov Chain Without Mixing*, 114 PROCS. NAT'L ACAD. SCIS. 2860 (2017).

33. *See League of Women Voters*, 178 A.3d at 779–80 (citation omitted).

remedial plan to be used in the 2018 primary and general congressional elections.[34] Partisan observers both credited and blamed the court-ordered redistricting with helping Pennsylvania Democrats gain three U.S. House seats in November 2018.[35]

Last year's developments in Wisconsin and Pennsylvania have set up *Rucho v. Common Cause*, a challenge to North Carolina's 2016 congressional redistricting plan, as a key test of whether the ensemble approach supplies the necessary analytical framework for an evidence-based adjudication of partisan gerrymandering claims under the United States Constitution.[36] *Rucho* came on the heels of litigation earlier this decade that ultimately resulted in the invalidation of two districts in the state's 2011 congressional plan as unconstitutional racial gerrymanders.[37] In the ensuing 2016 court-ordered redistricting, Republican state legislators commissioned and expressly directed a map-drawing expert to maintain the Republicans' 10–3 majority in the state's delegation.[38] Under the resulting remedial plan, North Carolina Republicans in November 2016 won ten of the state's thirteen United States House seats on 53.22 percent of the statewide popular vote.[39]

In *Rucho*, Common Cause, together with voters from each of the state's thirteen congressional districts and other plaintiffs,[40] brought claims against legislative and state defendants responsible for drafting and implementing the 2016 remedial plan for violations of Article I, the First Amendment, and

---

34. *See id.* at 825.

35. *See* David A. Lieb, *Election Shows How Gerrymandering is Difficult to Overcome*, ASSOCIATED PRESS (Nov. 17, 2018), https://www.apnews.com/3b4e63717b164dc19 9d02bd21aa17307 (reporting *League of Women Voters* plaintiff Bill Marx's comment that the November 2018 election resulted in "a more fair congressional representation of the will of the people in Pennsylvania" and state Republican Party spokesman Jason Gottesman's statement that "[t]he Pennsylvania Supreme Court robbed us of at least three to four congressional seats" by ordering the redistricting).

36. *See also* Charles & Fuentes-Rhower, *supra* note 21, at 237 (describing *Rucho* as a case that "combine[s] all of the issues presented in *Gill* and *Benisek* in a single case"). Oral argument in *Rucho v. Common Cause* has been set for March 2019. *See* Rucho v. Common Cause, No. 18-422 (S. Ct. Jan. 4, 2019) (jurisdiction postponed).

37. *See* Harris v. McCrory, 159 F. Supp. 3d 600, 627 (M.D.N.C. 2016), *aff'd sub nom.* Cooper v. Harris, 137 S. Ct. 1455, 1481–82 (2017).

38. *See* Common Cause v. Rucho, 318 F. Supp. 3d 777, 807–08 (M.D.N.C. 2018) (describing proposal and adoption of "Partisan Advantage" criterion that directed redistricting consultant Thomas Hofeller's drawing of the 2016 redistricting plan).

39. *See id.* at 810.

40. One lawsuit filed by Common Cause, the North Carolina Democratic Party, and fourteen voters and another filed by the League of Women Voters of North Carolina and twelve voters have been consolidated by the district court. *See id.* at 810–11.

the Equal Protection Clause of the Fourteenth Amendment.[41] At trial, one of us testified as an expert for Common Cause based on ensemble analyses performed by our research team centered at Duke University's Department of Mathematics.[42] After a four-day trial, a three-judge panel majority found for the plaintiffs on all claims and enjoined the 2016 plan's use in the 2018 elections.[43] The Supreme Court promptly stayed the district court's injunction,[44] however, and ultimately vacated and remanded the trial court's decision for reconsideration in light of its standing decision in *Gill*.[45] On remand, the district court reaffirmed its findings of constitutional violations,[46] but not in time for the plaintiffs to propose meaningful relief ahead of November.[47] Despite a national "blue wave,"[48] North Carolina Republicans managed to elect nine out of twelve representatives to the 116th Congress on a 50.39%–48.35% majority of the statewide popular vote, with one district's election results nullified amid charges of felony election fraud.[49] The court did enjoin the 2016 plan's use in any election *after* November 6, 2018.[50] The plans for North Carolina's 2020 congressional

---

41. *See id.* at 799.

42. *See id.* at 870–74 (describing testimony of Jonathan Mattingly); *see also* Sachet Bangia et al., *Redistricting: Drawing the Line*, ARXIV.ORG, https://arxiv.org/abs/1704.03360 (last updated May 8, 2017) (presenting ensemble analyses of 2016 North Carolina congressional plan).

43. *See* Common Cause v. Rucho, 279 F. Supp. 3d 587, 690 (M.D.N.C. 2018).

44. Rucho v. Common Cause, 138 S. Ct. 923 (2018) (mem.).

45. Rucho v. Common Cause, 138 S. Ct. 2679 (2018) (mem.).

46. *Rucho*, 318 F. Supp. 3d at 941.

47. Mem. Regarding Remedies from the Common Cause and League of Women Voters Plaintiffs at 5, Common Cause v. Rucho, 318 F. Supp. 3d 777 (M.D.N.C. 2018) (No. 1:16-CV-1026-WO-JEP (asserting that "every election conducted under an unconstitutional plan visits irreparable harm on voters," but concluding that "a statewide redistricting just weeks before Election Day would not be a good-government solution").

48. *See, e.g.*, B.J. Rudell, *2018 Midterms: A Blue Wave or Merely an Electoral Adjustment Into a New Presidency?*, HILL (Dec. 2, 2018, 5:00 PM), https://thehill.com/opinion/campaign/419308-2018-midterm-election-a-blue-wave-or-merely-an-electoral-adjustment-into-a (concluding that "no midterm election in the past century or more has been so lopsided" as 2018).

49. 2018 United States House of Representatives Elections in North Carolina, WIKIPEDIA, https://en.wikipedia.org/wiki/2018_United_States_House_of_Representatives_elections_in_North_Carolina (last visited May 7, 2019). The State Board of Elections has found that fraud invalidated the ninth district's results in the 2018 general election and has set dates for new primary and general elections in 2019. *See* Alan Blinder, *North Carolina Sets New Date for Redo Election in Congressional House Race*, N.Y. TIMES (Mar. 4, 2019), https://www.nytimes.com/2019/03/04/us/north-carolina-special-election-house-ninth.html.

50. *See Rucho*, 318 F. Supp. 3d at 942.

elections are therefore still at stake as the case returns on appeal to the Supreme Court this term.[51]

The aim of this Article is to highlight the significant and distinctive role ensemble methods are likely to play in the Supreme Court's forthcoming adjudication of *Rucho*. Three questions have been presented on appeal: (1) whether plaintiffs have standing to press their partisan gerrymandering claims; (2) whether plaintiffs' partisan gerrymandering claims are justiciable; (3) and whether North Carolina's 2016 congressional map is, in fact, an unconstitutional partisan gerrymander.[52] As we will aim to show in this Article, our ensemble analysis in *Rucho* has provided the Supreme Court with a powerful set of new analytical tools for adjudicating at least the first two of these questions. Further proceedings in *Rucho* will hold special importance for the many teams of mathematical scientists around the country who are continuing to develop and refine the ensemble approach as a statistically informed response to Justice Kennedy's call in *Vieth* for "new methods of analysis."[53] Ultimately, however, it remains for the Court to determine whether and how to apply these methodological advances to the third question on appeal; i.e., the formulation and application of constitutional standards in *Rucho* and in future cases.

The remainder of this Article is organized as follows. Part II reviews the search for a justiciable quantitative standard for partisan gerrymandering, contrasting the ensemble approach with previous efforts to quantify the effects of gerrymandering in terms of partisan symmetry measures.[54] Part III describes how the Duke team generated, analyzed, and presented the ensemble of North Carolina congressional plans at trial in *Rucho*.[55] Part IV reviews the district court's adjudication of our ensemble evidence in

---

51. *See* Rucho v. Common Cause, 139 S. Ct. 782 (2018) (mem.).

52. *See id.*

53. *See generally, e.g.*, Jowei Chen & Jonathan Rodden, *Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders*, 14 ELECTION L.J. 331 (2015); Jowei Chen & Jonathan Rodden, *Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures*, 8 Q.J. POL. SCI. 239 (2013) [hereinafter *Unintentional Gerrymandering*]; Maria Chikina et al., *Assessing Significance in a Markov Chain Without Mixing*, 114 PROC. NAT'L ACAD. SCI. 2860 (2017); Moon Duchin, Outlier Analysis for Pennsylvania Congressional Redistricting (Feb. 2018) (unpublished manuscript), https://sites.tufts.edu/vrdi/files/2018/06/md-report.pdf; Metric Geometry and Gerrymandering Grp., *Comparison of Districting Plans for the Virginia House of Delegates*, MGGG.ORG, https://mggg.org/VA-report.pdf (last visited Feb. 27, 2019); Benjamin Fifield et al., Benjamin Fifield et al., A New Automated Redistricting Simulator Using Markov Chain Monte Carlo (May 24, 2018) (unpublished manuscript), https://imai.fas.harvard.edu/research/files/redist.pdf.

54. *See infra* pp. 1249–57

55. *See infra* pp. 1258–68.

addressing the Supreme Court's standing and justiciability concerns on remand in the wake of *Gill*.[56] Part V concludes by discussing ongoing work to improve the ensemble approach for future adjudication of constitutional challenges to partisan gerrymandering.[57]

II. THE EMERGENCE OF THE ENSEMBLE APPROACH

A. The Search for a Justiciable Standard

Setting the stage for *Gill* was a line of Supreme Court caselaw that has preserved the justiciability of partisan gerrymandering claims in principle, but has rejected every proposed test for adjudicating such claims in practice. For example, in *Davis v. Bandemer*,[58] Indiana Democrats brought an equal protection challenge to legislative plans under which they won only 43 of 100 House seats despite receiving 51.9 percent of votes cast statewide.[59] The district court sustained the challenge.[60] Writing for a 6–3 majority, Justice White reasoned that the mutability of partisan affiliation might be relevant to the manner in which a partisan gerrymandering claim is adjudicated under equal protection doctrine, but does not distinguish such a claim from racial gerrymandering claims in terms of justiciability.[61]

In a plurality part of the opinion, Justice White elaborated on the basic requirements of discriminatory intent and effect for an equal protection challenge in the context of partisan gerrymandering.[62] In affirming the trial court's intent finding,[63] Justice White observed that "districting inevitably has and is intended to have substantial political consequences," and even where drafters work "'with census, not political, data and achieve population equality without regard for political impact,'" any resulting discriminatory effect is ultimately intentional.[64] The discriminatory effect requirement, on

---

56. *See infra* pp. 1269–75. A review of the entire remand opinion in *Rucho* is beyond the scope of this Article.

57. *See infra* p. 1275–76.

58. 478 U.S. 109 (1986).

59. *See id.* at 115.

60. Bandemer v. Davis, 603 F. Supp. 1479, 1495–96 (S.D. Ind. 1984).

61. *Davis*, 478 U.S. at 125.

62. *See id.* at 127 (citing Mobile v. Bolden, 446 U.S. 55, 67–68 (1980)) ("We also agree with the District Court that in order to succeed the Bandemer plaintiffs were required to prove both intentional discrimination against an identifiable political group and an actual discriminatory effect on that group.").

63. *See id.* at 127.

64. [T]his politically mindless approach may produce, whether intended or not, the most grossly gerrymandered results; and, in any event, it is most unlikely that the political impact of such a plan would remain undiscovered by the time it

the other hand, could not be satisfied "by the simple fact of an apportionment scheme that makes winning elections more difficult" or "a failure of proportional representation alone."[65] Instead, wrote Justice White, "unconstitutional discrimination occurs only when the electoral system is arranged in a manner that will consistently degrade a voter's or a group of voters' influence on the political process as a whole."[66] Against this standard, Justice White criticized the trial court's effect findings, which had been based primarily on the results of the 1982 elections,[67] noting that "[r]elying on a single election to prove unconstitutional discrimination is unsatisfactory."[68] He also found reversible error in the lack of any finding that the challenged plans "would consign the Democrats to a minority status in the Assembly throughout the 1980's or that the Democrats would have no hope of doing any better in the reapportionment that would occur after the 1990 census."[69]

Next, in *Vieth v. Jubelirer*,[70] Pennsylvania voters brought an equal protection challenge to a redistricting plan under which Republicans allegedly held voting majorities in thirteen of Pennsylvania's nineteen U.S. House districts even though Democrats outnumbered Republicans in the state.[71] The district court dismissed the challenge as "simply an argument for proportional representation," which had been rejected in *Bandemer* as a basis for an equal protection claim.[72] On appeal to the Supreme Court, the plaintiffs argued that discriminatory intent could be proven by a showing "by direct evidence or by circumstantial evidence that other neutral and legitimate redistricting criteria were subordinated to the goal of achieving partisan advantage."[73] The plaintiffs further argued that discriminatory effect could be shown when "(1) the plaintiffs show that the districts systematically 'pack' and 'crack' the rival party's voters, and (2) the court's examination of the 'totality of circumstances' confirms that the map can thwart the plaintiffs' ability to translate a majority of votes into a majority of seats."[74]

---

was proposed or adopted, in which event the results would be both known and, if not changed, intended.

Id. at 129 (quoting Gaffney v. Cummings, 412 U.S. 735, 753 (1973)).

65. *Id.* at 132 (citing *Mobile*, 446 U.S. at 111 n.7).
66. *Id.* at 132.
67. *Id.* at 134.
68. *Id.* at 135.
69. *Id.* at 135–36.
70. 541 U.S. 267 (2004).
71. *See* Vieth v. Pennsylvania, 188 F. Supp. 2d 532, 546 (M.D. Pa. 2002).
72. *See id.*
73. *Vieth*, 541 U.S. at 284.
74. *Id.* at 286 (citation omitted).

A 5–4 majority voted to affirm the judgment, with Justice Scalia authoring a plurality opinion, Justice Kennedy writing a separate concurrence, and Justices Stevens, Souter, and Breyer writing separate dissents in which each proposed a different standard for adjudication.[75] Writing for four Justices, Justice Scalia questioned whether the plaintiffs' intent test required that partisan intent "outweigh" all other redistricting goals and how such "'outweighing' [is] to be determined."[76] He also criticized the proposed effect standard as relying on a nonexistent group right to proportional representation.[77] Scalia proceeded to conclude from the eighteen-year absence of judicially discernible and manageable standards for partisan gerrymandering claims that none would ever be forthcoming.[78]

Justice Kennedy's concurrence in *Vieth*, however, took a longer view regarding the potential emergence of "suitable standards with which to measure the burden a gerrymander imposes on representational rights."[79] He identified:

> [T]he "rapid evolution of technologies in the apportionment field" as presenting both a potential "threat" in the hands of those who would "use partisan favoritism in districting in an unconstitutional manner," and a potential "promise" as a source of "new methods of analysis . . . [t]hat would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards."[80]

Kennedy further opined that liability for partisan gerrymandering "must rest on something more than the conclusion that political classifications were applied," and "must rest instead on a conclusion that the classifications, though generally permissible, were applied in an invidious manner or in a way unrelated to any legitimate legislative objective."[81]

In *League of United Latin American Citizens v. Perry*[82] (*LULAC*), Texas interest groups brought equal protection and First Amendment challenges to an unusual mid-decade redistricting plan that was allegedly "driven solely

75. *Id.* at 269–70.
76. *Id.* at 287.
77. *Id.* at 269.
78. *Id.* at 281.
79. *Id.* at 313 (Kennedy, J., concurring).
80. *Id.* at 312–13.
81. *Id.* at 307.
82. 548 U.S. 399 (2006).

by a partisan agenda."[83] On remand after *Vieth*, the plaintiffs asked the district court to "distill from the *Vieth* opinions" the rule that a mid-decade redistricting is unconstitutional "when the evidence makes clear that the legislature was driven solely by a partisan agenda."[84] The district court declined to do so and dismissed the challenges.[85] On appeal, Justice Kennedy wrote an opinion announcing the judgment of the Court[86] in which he also provided his separate views regarding the plaintiffs' proposed test.[87] For Justice Kennedy, the test's lack of an effect prong was fatal in that a partisan gerrymandering claim "must do what appellants' sole-motivation theory explicitly disavows: show a burden, as measured by a reliable standard, on the complainants' representational rights."[88] Kennedy pointed out that the Texas plan appeared to have less of a discriminatory effect than the plan that survived constitutional scrutiny in *Vieth*, at least by the "rough measure" of vote-seat parity.[89]

In finding the lack of a reliable standard for discriminatory effect in *LULAC*, Justice Kennedy also considered a partisan bias standard proposed in political scientist Gary King's *amicus* brief.[90] King's standard measures the partisan bias of a plan in terms of "the extent to which a majority party would fare better than the minority party, should their respective shares of the vote reverse."[91] Justice Kennedy suggested the proposed standard, while potentially informative, would not be judicially discoverable and manageable:

> Even assuming a court could choose reliably among different models of shifting voter preferences, we are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs. . . . Without altogether discounting its utility in redistricting planning and

---

83. Henderson v. Perry, 399 F. Supp. 2d 756, 762 (E.D. Tex. 2005).

84. *Id.* at 764.

85. *Id.* at 778.

86. *League of United Latin Am. Citizens*, 548 U.S. at 408.

87. *Id.* (citing Part II-C of the opinion).

88. *Id.* at 418.

89. *Id.* at 419 ("To be sure, there is no constitutional requirement of proportional representation, and equating a party's statewide share of the vote with its portion of the congressional delegation is a rough measure at best.").

90. *See id.* at 419–20.

91. *Id.* at 420 (citing Br. of Amici Curiae Professor Gary King et al. at 5, League of United Latin Am. Citizens v. Perry, 548 U.S. 399 (2006) (Nos. 05-204, 05-254, 05-276, 05-439)).

> litigation, I would conclude asymmetry alone is not a reliable measure of unconstitutional partisanship.[92]

Other Justices took a more positive view of King's proposal. In dissenting from Justice Kennedy's rejection of the plaintiffs' proposed test, Justices Stevens and Souter suggested the possibility that some partisan symmetry standard could serve as a reliable measure of discriminatory effect in a future case.[93]

The Court's rejections of the plaintiffs' partisan gerrymandering standards in *Bandemer*, *Vieth*, and *LULAC* led the plaintiffs in *Gill* to frame their case around a partisan symmetry measure that (1) did not appeal to an unrecognized principle of proportional representation, and (2) was calculated from actual, not hypothetical, election results.[94] The next section discusses some of the difficulties of this approach.

B. The Focus on Partisan Symmetry

Supported by well-established political science literature,[95] the *Gill* plaintiffs followed the *LULAC* plaintiffs in seeking to frame their partisan gerrymandering claims against a baseline of partisan symmetry; i.e., the

---

92. *Id.* at 420 (citation omitted).

93. *See id.* at 466–67 (Stevens, J., concurring in part and dissenting in part) (describing partisan symmetry as "undoubtedly 'a reliable standard' for measuring a 'burden . . . on the complainants' representative rights'" and finding the measure sufficient to show discriminatory effect in the instant case (citation omitted)); *id.* at 468 n.9 (Stevens, J., concurring in part and dissenting in part) ("I appreciate Justice Kennedy's leaving the door open to the use of the [partisan symmetry] standard in future cases."); *id.* at 483–84 (Souter, J., concurring in part and dissenting in part) ("[N]or do I rule out the utility of a criterion of symmetry as a test . . . . Interest in exploring this notion is evident . . . . Perhaps further attention could be devoted to the administrability of such a criterion at all levels of redistricting and its review." (citations omitted)).

94. *See id.* at 420 (rejecting a proposed partisan symmetry standard that would "invalidate[] a map based on unfair results that would occur in a hypothetical state of affairs"); Vieth v. Jubelirer, 541 U.S. 267, 284–88 (2004) (plurality opinion) (rejecting a proposed discriminatory effect standard as "rest[ing] upon the principle that groups (or at least political-action groups) have a right to proportional representation"); Davis v. Bandemer, 478 U.S. 109, 129–32 (1986) (rejecting a proposed discriminatory effect requirement that could be satisfied by "a failure of proportional representation alone . . . .").

95. *See generally* Robert X. Browning & Gary King, *Seats, Votes, and Gerrymandering: Estimating Representation and Bias in State Legislative Redistricting*, 9 LAW & POL'Y 305 (1987); Bernard Grofman, *Measures of Bias and Proportionality in Seats-Votes Relationships*, 9 POL. METHODOLOGY 295 (1983); Gary King & Robert X. Browning, *Democratic Representation and Partisan Bias in Congressional Elections*, 81 AM. POL. SCI. REV. 1251 (1987); Edward R. Tufte, *The Relationship Between Seats and Votes in Two-Party Systems*, 67 AM. POL. SCI. REV. 540 (1973).

expectation that the two major political parties should each require the same statewide number of popular votes to win a given number of legislative seats.[96] When *LULAC amici* Bernard Grofman and Gary King had proposed a constitutional standard based on a partisan symmetry measure, they found no takers, although five Justices declined to rule out the use of partisan symmetry as part of a broader test.[97] A decade later, the *Gill* plaintiffs presented the efficiency gap, a different partisan symmetry measure intended in large part to address Justice Kennedy's misgivings about Grofman and King's proposed standard in *LULAC*.[98] The efficiency gap compares both major parties' votes "wasted"—either in a losing cause or in excess of the number needed to win—across all districts, and is therefore sensitive at the margins to one party's efforts to "crack" the other party's supporters into sizeable but losing minorities across multiple districts—or to "pack" them into overwhelming majorities in a few districts.[99] As originally conceived by Nicholas Stephanopoulos and *Gill amicus* Eric McGhee, the efficiency gap could provide quantitative thresholds for the constitutionality of legislative districting plans.[100]

The problem with using partisan symmetry as a normative baseline is that actual partisan symmetry is rare. The concentration of Democrats in more urban areas—and the complexities of political geography—are usually enough to ensure that most district plans are not a level playing field for Democrats and Republicans, even when drawn by independent commissions and/or without reference to partisan voter information.[101] For example,

---

96. Gill v. Whitford, 138 S. Ct. 1916, 1933 (2018).

97. *See* NICHOLAS R. SEABROOK, DRAWING THE LINES: CONSTRAINTS ON PARTISAN GERRYMANDERING IN U.S. POLITICS 58–59 (2017) (noting that the LULAC Court "reject[ed] partisan symmetry as a standard for adjudicating political gerrymandering," but "expressed considerable encouragement for its potential use in the future as part of a broader test"); *see also* Bernard Grofman & Gary King, *The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering After* LULAC v. Perry, 6 ELECTION L.J. 2, 4 (2007) (emphasizing the justices' "considerable positive attention" to the authors' proposed criterion).

98. *See League of United Latin Am. Citizens*, 548 U.S. at 420 (noting that Grofman and King's partisan asymmetry measure "may in large part depend on conjecture about where possible vote-switchers will reside"); Stephanopoulos & McGhee, *supra* note 15, at 849, 896 (introducing the efficiency gap as "a new measure of partisan symmetry" and arguing that it "avoids the need to estimate hypothetical election results (and, with it, the need to speculate about vote switchers' locations).").

99. *See* Stephanopoulos & McGhee, *supra* note 15, at 851–52 (relating the efficiency gap to the measurement of cracking and packing strategies).

100. *See id.* at 884–85.

101. *See* Nicholas Goedert, *Gerrymandering or Geography? How Democrats Won the Popular Vote But Lost the Congress in 2012*, 1 RES. & POL. 1, 1–2 (2014) (observing that "the current geographic distribution of partisans now leaves Democrats at a disadvantage so long as

Gill's lead plaintiff William Whitford was "naturally" packed in his district with other Madison Democrats, a situation that favored Republicans even in the plaintiffs' demonstration plan.[102] Such urban concentrations of Democrats in Madison and Milwaukee led the trial court to find that "Wisconsin's political geography affords Republicans a modest natural advantage in districting,"[103] although it ultimately concluded that this natural tilt was much smaller than the "large partisan effect" of the challenged gerrymander.[104] In focusing on standing, the Supreme Court did not review these findings. But Chief Justice Roberts's characterization of the efficiency gap as "sociological gobbledygook"[105] at oral argument bespoke his reluctance to weigh maps on a scale whose values had no meaning to "the intelligent man on the street,"[106] let alone one whose baseline of zero had no corresponding real-world benchmark.[107]

---

congressional representation is based on contiguous geographic districts" and surveying previous studies of this phenomenon).

102. *See* Gill v. Whitford, 138 S. Ct. 1916, 1933 (2018).

103. Whitford v. Gill, 218 F. Supp. 3d 837, 919–20 (W.D. Wis. 2016) (attributing this advantage largely to the fact that "Democratic voters are uniquely packed in urban centers like Milwaukee and Madison").

104. *Id.* at 926.

105. Tr. of Oral Arg. at 40, Gill v. Whitford, 137 S. Ct. 2289 (Oct. 3, 2017) (No. 16-1161) [hereinafter Gill Oral Argument].

106. *Id.* at 37.

107. The president of the American Sociological Association and other observers took Chief Justice Roberts to be casting aspersions on all of quantitative social science. *See* Letter from Eduardo Bonilla-Silva, President, Am. Sociological Ass'n, to Chief Justice John Roberts (Oct. 10, 2017), http://www.asanet.org/news-events/asa-news/asa-president-eduardo-bonilla-silva-responds-chief-justice-john-roberts (describing Roberts's comment as an admission of a "lack of understanding of social science"); Philip Rocco, *Justice Roberts Said Political Science is 'Sociological Gobbledygook.' Here's Why He Said It, and Why He's Mistaken*, WASH. POST (Oct. 4, 2017), https://www.washingtonpost.com/news/monkey-cage/wp/2017/10/04/justice-roberts-said-political-science-is-sociological-gobbledygook-heres-why-he-said-it-and-why-hes-mistaken/?noredirect=on&utm_term=.635a9d642d13 ("In short, [Roberts] is concerned that voters will not trust judicial decisions based on social science."); *but see* Ed Whelan, *Speaking of 'Sociological Gobbledygook'*, NAT'L REV. (Oct. 11, 2017, 3:10 PM), https://www.nationalreview.com/blog/bench-memos/roberts-sociological-gobbledygook (criticizing Bonilla-Silva for "pretending that the Chief was dismissing *all* of sociology as 'gobbledygook.'"). The Chief Justice actually was being more specifically critical of the unreliability of election predictions based on statistical measures of partisan advantage that had previously been before the Court:

> [The plaintiffs in] *Bandemer* predicted the Democrats would never be able to attain a majority. It was 50/50 the next election, and they got a majority the one after that. You already mentioned *Vieth*. It was five days, right, after the District Court said . . . [the] Republicans are never going to get elected. And they won every single race. Predicting on the basis of the statistics that are before us has been a very hazardous enterprise.

C. The Motivation for the Ensemble Approach

Partisan symmetry measurements compare the abilities of parties to translate statewide votes into seats against a baseline assumption of parity in electoral performance. Alternative explanations for deviations from this baseline—including differences in the political geography of different states as measured by the results of elections for different offices on different dates—can confound the use of partisan symmetry measures as general-purpose tools to identify and quantify the effects of partisan gerrymandering.[108]

Rather than comparing seats to votes, we instead compare a large collection (an ensemble) of redistricting plans to an enacted plan. The ensemble of plans reveals—and discovers—the asymmetric baseline votes-to-seats relationships that partisan symmetry measures simply assume away. There are no *a priori* judgments about how votes should translate into seats; instead the ensemble's plans are randomly generated subject to a stipulated set of legal criteria for compliant redistricting plans. Such criteria may be imposed by the United States Constitution,[109] the Voting Rights Act,[110] state constitutions,[111] state statutes,[112] committee resolutions,[113] and court

---

Gill Oral Argument, *supra* note 105, at 48–49.

108. *See generally* Vieth v. Jubelirer, 541 U.S. 267, 286–90 (2004) (rejecting test for discriminatory effects that looks to whether "the map can thwart the plaintiffs' ability to translate a majority of votes into a majority of seats" as non-justiciable in light of these complications).

109. *See* Karcher v. Daggett, 462 U.S. 725, 730–31 (1983) (requiring that any significant variance in population among districts be necessary to achieve a legitimate state purpose).

110. *See* Thornburg v. Gingles, 478 U.S. 30, 50–51 (1986) (describing a three-pronged structural test for when "multimember districts . . . operate to impair minority voters' ability to elect representatives of their choice"); Growe v. Emison, 507 U.S. 25, 39–40 (1993) (extending the applicability of the *Gingles* test to single-member districting).

111. *See, e.g.*, WIS. CONST., art. IV, § 4 (permitting deviation from population equality to accommodate traditional districting objectives, requiring assembly districts "to be bounded by county, precinct, town or ward lines, to consist of contiguous territory and be in as compact form as practicable," and senate districts to consist of "convenient contiguous territory" comprised of undivided assembly districts).

112. *See, e.g.*, VA. CODE ANN. § 24.2-305 (2001) ("Each election district and precinct shall be composed of compact and contiguous territory and shall have clearly defined and clearly observable boundaries.").

113. *See, e.g.*, JOINT REAPPORTIONMENT COMM., Committee Resolution: Third Congressional District Criteria (Aug. 17, 2015) (adopting criteria of population equality, Voting Rights Act compliance, contiguity, compactness, and preserving communities of interest for the redrawing of Virginia's Third Congressional District to comply with a court order).

orders.[114] The ensemble is small in comparison with the astronomical number of compliant plans that could have been drawn[115] but may still be deemed representative in the sense that generating many more plans will have, at most, a negligible effect on the quantitative and qualitative conclusions.[116]

Since all of the ensemble's plans overlay the same political geography as the challenged plan, many of the ensemble plans will exhibit partisan effects from natural packing and cracking similar to those in the challenged plan, while deliberately packed and cracked districts in the challenged plan will have no counterpart in the ensemble. In this way, ensemble analysis serves to separate out the effects of political geography from the specific partisan features of the challenged plan.[117] If the challenged plan's partisan effects are extreme outliers in comparison with those of the plans in the ensemble, this finding can serve as evidence that some consideration other than compliance with stated criteria was involved in the redistricting process, and can help quantify the vote-diluting effects of cracking and packing in individual districts and statewide.[118] In the next Part, we will explain how the Duke team generated and analyzed the ensemble of plans presented in *Rucho*.

---

114. *See, e.g.*, Harris v. McCrory, 159 F. Supp. 3d 600, 627 (M.D.N.C. 2016), *aff'd sub nom.* Cooper v. Harris, 137 S. Ct. 1455, 1481–82 (2017) (requiring legislature to draw a new congressional redistricting plan under which race-based redistricting must satisfy strict scrutiny).

115. *See generally* Moon Duchin, *Geometry v. Gerrymandering*, SCI. AM., Nov. 2018, at 49, 51 (noting that there are more than 700 trillion ways of partitioning a 9×9 grid into nine contiguous districts of equal size).

116. *See infra* notes 156–53 and accompanying text.

117. *See* Gregory Herschlag et al., *Evaluating Partisan Gerrymandering in Wi*sconsin 5 (Sept. 7, 2017) (unpublished manuscript), https://arxiv.org/pdf/1709.01596.pdf (explaining that ensemble "analysis allows us to separate out the effect of the geopolitical landscape, and to show that the [challenged] Act 43 map generates extreme partisan asymmetry above and beyond this effect"); *cf.* Gonzalez v. City of Aurora, Ill., 535 F.3d 594, 599–600 (7th Cir. 2008) (Easterbrook, C.J.) (opining that if racial gerrymandering plaintiffs had submitted 1,000 computer-generated "random, race-blind" maps showing that the challenged plan was an outlier with respect to the number of "'Latino effective' districts . . . [t]hen a court might sensibly conclude that [defendants] had diluted the Latino vote by undermining the normal effects of the choices that Aurora's citizens had made about where to live.").

118. *See* Bangia et al., *supra* note 42, at 2 (describing gerrymandering index and representativeness index as "measures of gerrymandering where the effects of packing . . . and cracking . . . can be better identified"); *id.* at 5 (explaining that gerrymandering index "quantifies how packed or depleted the collection of districts is relative to what is expected from the ensemble of 'reasonable' redistrictings we have created").

III. THE ENSEMBLE AT TRIAL

A. Procedural History

Rucho v. Common Cause[119] is a partisan gerrymandering challenge to the 2016 remedial plan that resulted from an earlier racial gerrymandering challenge to North Carolina's decennial congressional redistricting.[120] In 2011, after winning control of both houses of the General Assembly in 2010, Robert Rucho and David Lewis, the Republican chairs of the Senate and House redistricting committees, respectively, engaged the Republican National Committee's redistricting coordinator Thomas Hofeller to redraw the state's congressional map.[121] The legislators privately instructed Hofeller that the plan's primary goal was "to create as many districts as possible in which GOP candidates would be able to successfully compete for office."[122] The legislators directed Hofeller to "creat[e] new majority African American districts,"[123] ostensibly to comply with section two of the Voting Rights Act,[124] but effectively reduced the influence of African American voters by packing more of them into Districts 1 and 12.[125] Under the resulting plans, Republicans won 9–4 and 10–3 majorities of the state's United States House seats in 2012 and 2014, respectively.[126] In the meantime, African American voters in these districts sued in the Middle District of North Carolina to challenge the 2011 plan as a racial gerrymander in violation of the Equal Protection Clause of the Fourteenth Amendment.[127] In *Cooper v. Harris*, the Supreme Court affirmed the district court's conclusions that racial considerations predominated in the drawing of Districts 1 and 12 and were not sufficiently warranted by Voting Rights Act concerns to survive strict scrutiny,[128] leaving in place a February 5, 2016 order for the General Assembly to draw a new congressional district plan.[129]

---

119. Common Cause v. Rucho, Nos. 1:16-CV-01026, 1:16-CV-1164 (M.D.N.C. 2018), *stayed pending appeal sub nom.* Rucho v. Common Cause, No. 17A745 (S. Ct. 2018).

120. *See* Common Cause v. Rucho, 318 F. Supp. 3d 777, 810 (M.D.N.C. 2018).

121. *See* Harris v. McCrory, 159 F. Supp. 3d 600, 607 (M.D.N.C. 2016). Redistricting in North Carolina is exclusively within the power of the legislative branch and not subject to veto by the governor. *See* N.C. CONST., art. II, § 22(5)(b)–(d).

122. *Rucho*, 318 F. Supp. 3d at 803.

123. *See Harris*, 159 F. Supp. 3d at 608.

124. *See id.*

125. *See id.* at 609.

126. *See Rucho*, 318 F. Supp. 3d at 804.

127. *See id.* (citing *Harris*, 159 F. Supp. 3d at 609–10).

128. *See* Cooper v. Harris, 137 S. Ct. 1455, 1468, 1474 (2017).

129. *Harris*, 159 F. Supp. 3d at 627.

With Republicans still in control of the General Assembly, Rucho and Lewis again hired Thomas Hofeller, this time to draw the 2016 remedial plan.[130] The legislators privately instructed Hofeller to use precinct-level election results from statewide elections, excluding presidential elections, to ensure that the resulting plan would maintain the 10–3 Republican majority in the state's congressional delegation.[131] By February 13, 2016, Hofeller presented Rucho and Lewis with a "near-final" version of the enacted plan,[132] which Lewis confirmed would yield the intended 10–3 partisan advantage.[133] It was only after this *fait accompli* that the redistricting committee that Rucho and Lewis co-chaired received public comments and voted on a set of criteria that purportedly were to govern the redistricting process.[134] On February 16, the committee adopted criteria authorizing the use of election data to support "reasonable efforts to construct districts . . . to maintain the current partisan makeup of North Carolina's congressional delegation" along party lines,[135] thereby effectively ratifying Rucho and Lewis's earlier private instructions to Hofeller. Over the next three days, the committee, the Senate, and the House all adopted Hofeller's plan, with slight revisions,[136] along party lines.[137] Reflecting the openly partisan redistricting process, Lewis stated during the House debate, "I think electing Republicans is better than electing Democrats. So I drew this map to help foster what I think is better for the country."[138]

In August and September 2016, two groups of plaintiffs led by Common Cause and the League of Women Voters of North Carolina sued Rucho, Lewis, and other state officials in the Middle District of North Carolina, challenging the 2016 remedial plan as a partisan gerrymander under the First and Fourteenth Amendments and Article I, Sections 2 and 4 of the United States Constitution.[139] The district court consolidated the cases and conducted a four-day bench trial in October 2017.[140]

---

130. *Rucho*, 318 F. Supp. 3d. at 805.

131. *See id.*

132. *See id.* at 806–07. The Senate Redistricting Committee revised Hofeller's plan by shifting two whole precincts and one partial precinct to avoid districting two incumbents together. *See id.* at 809 n.6.

133. *See id.* at 807.

134. *See id.* at 807–08.

135. *Id.* at 807.

136. *See id.* at 806–07, 809 n.6.

137. *Id.* at 809.

138. *Id.*

139. *See id.* at 810–11.

140. *See id.* at 811.

B. Generating the North Carolina Ensemble

In analyzing North Carolina's 2016 congressional redistricting for presentation at trial, the Duke team aimed to generate a large and diverse set of legislative plans that complied with a stated set of redistricting criteria. These nonpartisan criteria were necessarily different from those actually adopted by the North Carolina General Assembly's redistricting committee,[141] which included a criterion expressly requiring "reasonable efforts to construct districts . . . to maintain the current [10–3] partisan makeup of North Carolina's congressional delegation"[142] and resulted in an openly partisan redistricting process.[143] Instead, the Duke team's ensemble used criteria taken from a 2015 bipartisan House bill to establish a nonpartisan commission and process for legislative and congressional redistricting.[144] Even though this bill died in committee,[145] we regard its

---

141. *See generally* N.C. GEN. ASSEMB. JOINT SELECT COMM. ON CONGRESSIONAL REDIST., 2016 CONTINGENT CONGRESSIONAL PLAN COMMITTEE ADOPTED CRITERIA (Feb. 16, 2016).

142. *Id.*

143. During floor debate on the enacted plan, State Rep. David Lewis, co-chair of the Joint Select Committee on Congressional Redistricting, stated, "I think electing Republicans is better than electing Democrats. So I drew this map to help foster what I think is better for the country." *Rucho*, 318 F. Supp. 3d at 809 (citation omitted).

144. *See* H.B. 92, 2015 N.C. Gen. Assemb., 2015–16 Sess. (N.C. 2015). These criteria, which were to be codified at N.C. GEN. STAT. § 120-4.54, included:

> (c) Congressional districts shall each have a population as nearly equal as practicable to the ideal population, but in all cases within one-tenth of a percent (0.1%) of the ideal population.
>
> (d) Legislative and congressional districts shall be drawn in a manner that complies with requirements of federal and State law.
>
> (e) To the extent consistent with other standards provided by this section, . . . [t]he number of counties and cities divided among more than one district shall be as small as possible, but in the case of cities located in more than one county, minimizing the division of counties prevails. The division of VTDs [voting tabulation districts] shall also be minimized consistent with the other standards of this section.
>
> (f) Districts shall be composed of convenient contiguous territory. Areas which meet only at the points of adjoining corners are not contiguous.
>
> (g) Districts shall be reasonably compact in form, to the extent consistent with the standards established by this section. In general, reasonably compact districts are those which are square, rectangular, or hexagonal in shape, and not irregularly shaped, to the extent of natural or political boundaries or those of VTDs. If it is necessary to compare the relative compactness of two or more districts, or of two or more alternative districting plans, [preference shall be given to the plan comprised of districts whose maximum north-south and east-west dimensions are, in the aggregate, most nearly equal and whose total perimeter is smallest].
>
> (h) No district shall be drawn for the purpose of favoring a political party, incumbent legislator, or member of Congress, or other person or group, or for the

provisions as representative of the longer-term process of deliberative policy formation in North Carolina around nonpartisan redistricting criteria.[146]

The algorithmic methods used in building an ensemble of plans require an often diverse and disparate set of operative redistricting criteria[147] to be expressed in precise mathematical terms. Thus, for example, in implementing the proposed nonpartisan criteria, the Duke team had to interpret the statutory term "irregularly shaped" and define just how "irregular" one will allow a shape to be before it is no longer compliant.[148] In principle, the range of typical behavior in an ensemble may change as this threshold changes, and this must be checked when generating ensembles.[149] Beyond stated legal requirements and guidelines, a number of implicit redistricting criteria may also require consideration. For example, the *Rucho*

> purpose of augmenting or diluting the voting strength of a language or racial minority group. In establishing districts, no use shall be made of any of the addresses or geographic locations of incumbents. Except to the extent required by [applicable law], no use shall be made of: (1) [p]olitical affiliations of registered voters; (2) [p]revious election results; [or] (3) [d]emographic information, other than population head counts.

Id.

145. There was no action on the bill following its referral to the House Committee on Elections. *See id.*

146. *See* H.B. 824, 2011 N.C. Gen. Assemb., 2011–12 Sess. (N.C. 2011) (proposing goals of equal population, legal compliance, preservation of political boundaries, contiguity, and length-width and perimeter compactness, and excluding goals of favoring parties, incumbents or racial groups and consideration of incumbents' residences); H.B. 674, 2017 N.C. Gen. Assemb., 2017–18 Sess. (N.C. 2011) (proposing goals of compactness, equal population, and preservation of political boundaries and communities of interest, and excluding consideration of party affiliation, past election results, incumbents' residences, and racial data except for purposes of legal compliance); *see also* Bangia et al., *supra* note 42, at 4 (describing H.B. 92 as "just the latest in a chain of bills which have been introduced over the years with similar criteria and aims").

147. *See supra* notes 109–14 and accompanying text.

148. *See* Gregory Herschlag et al., Quantifying Gerrymandering in North Carolina 9 (Jan. 9, 2018) [hereinafter Quantifying Gerrymandering in NC] (unpublished manuscript), https://arxiv.org/abs/1801.03783 (explaining that ensemble "analysis allows us to separate out the effect of the geopolitical landscape, and to show that the [challenged] Act 43 map generates extreme partisan asymmetry above and beyond this effect"); *see also* Daniel D. Polsby & Robert D. Popper, *The Third Criterion: Compactness as a Procedural Safeguard Against Partisan Gerrymandering*, 9 YALE L. & POL'Y REV. 301, 348–49 (1991) (proposing isoperimetric measure of compactness); Richard H. Pildes & Richard G. Niemi, *Expressive Harms, "Bizarre Districts," and Voting Rights: Evaluating Election-District Appearances After* Shaw v. Reno, 92 MICH. L. REV. 483, 555–56 & n.203 (1993) (endorsing Polsby-Popper score).

149. The analytical conclusions derived from the North Carolina ensemble were robust against changes in the compactness threshold. *See* Quantifying Gerrymandering in NC, *supra* note 148, at 18. However, it is unknown if this will always be the case.

defendants challenged some ensemble plans in which a district line traversed a county line more than once, a feature that has not appeared in recent enacted plans even though it is not legally prohibited.[150]

Having precisely chosen redistricting criteria—or a set of various redistricting criteria—one may then ask what a typical plan would look like given these criteria. Although one could, in principle, enumerate all possible redistricting plans that comply with the criteria, in most cases it is computationally infeasible, if not impossible, to carry out such an enumeration, and in fact unnecessary.[151] There are well-established statistical methods, collectively referred to as Markov Chain Monte Carlo techniques, for producing a representative ensemble of plans. These techniques are prolific across many areas of science, including image processing, molecular systems, genomics, and climate science, and have given scientists the ability to answer previously unanswerable questions.[152]

The basic idea of using Markov Chain Monte Carlo techniques to generate an ensemble of redistricting plans is to begin with some initial plan and perform a sequence of random steps, each of which incrementally changes the configuration of districts.[153] The possible kinds of incremental changes are defined *a priori*. In creating the North Carolina ensemble, the Duke team defined a step as the reassignment of an arbitrary vote tabulation district (VTD) on the boundary of a congressional district to an adjacent district.[154] The chance of taking one step as opposed to any other is characterized so that (1) the steps will often lead to compliant plans and (2) the sequence of steps will sometimes produce a drastic departure from the initial plan so that any possible redistricting plan may be reached. As shown in Figure 1, the sequence of steps make up a "random walk," and compliant plans along the random walk make up the ensemble of plans. This process can be made mathematically precise and is theoretically capable of sampling the entire space of redistricting plans.[155] Empirical tests can also provide practical validation that (1) the ensemble is independent of the initial redistricting plan and that (2) the ensemble of plans is representative, which

---

150. *See* Def.'s Proposed Findings of Fact and Conclusions of Law at 59–60, Common Cause v. Rucho, 318 F. Supp. 3d 777, 807–08 (M.D.N.C. 2018) (No. 1:16-CCV-1026-WO-JEP), https://www.brennancenter.org/sites/default/files/legal-work/CC_LWV_DefendantsProposedFindingsofFact.pdf.

151. *See supra* note 115 and accompanying text.

152. *See* Christian Robert & George Casella, *A Short History of Markov Chain Monte Carlo: Subjective Recollections from Incomplete Data*, 26 STAT. SCI. 102, 106–11 (2011).

153. *See id.*

154. *See, e.g.*, Quantifying Gerrymandering in NC, *supra* note 148, at 8.

155. *But see* Bangia et al., *supra* note 42, at 27 (noting that the authors were unable to explore the entire space of redistrictings, perhaps because of premature system cooling).

is to say that the conclusions drawn from the ensemble will not change if more plans were to be gathered.[156]

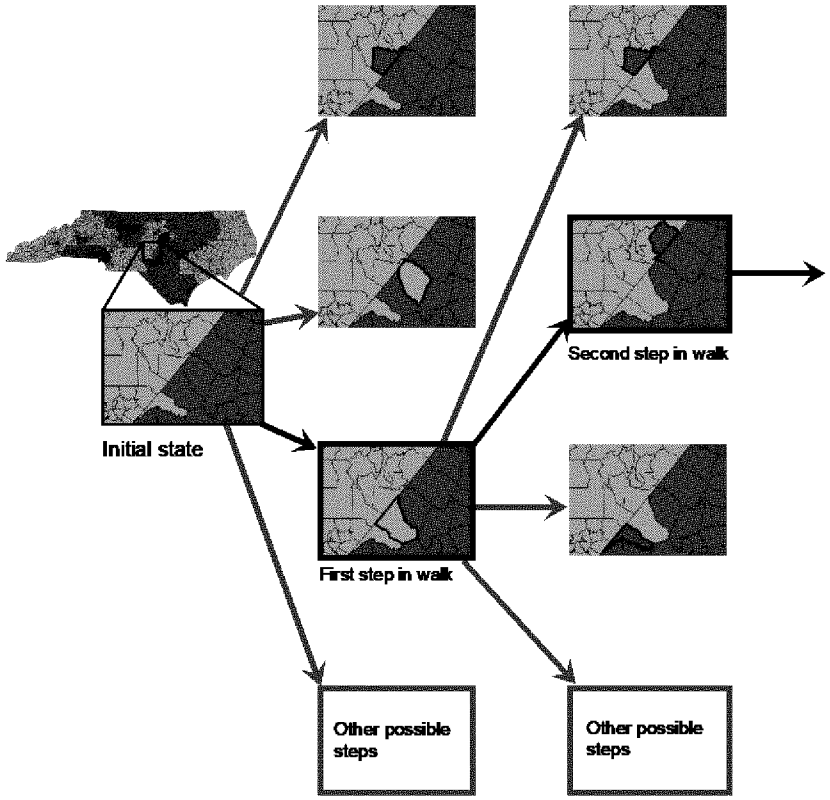

Figure 1. The initial plan is altered by changing VTD district assignments. Once many of these small changes are made, the districting plan is entirely different from where it began. The new plan periodically is examined; if it is compliant with the redistricting criteria, it is added to the ensemble.

156. *See* Andrew Gelman & Donald B. Rubin, *Inference from Iterative Simulation Using Multiple Sequences*, 7 STAT. SCI. 457, 458 (1992) (providing statistical test); Benjamin Fifield et al., A New Automated Redistricting Simulator Using Markov Chain Monte Carlo (May 24, 2018) (unpublished manuscript), https://imai.fas.harvard.edu/research/files/redist.pdf (illustrating application of statistical test to redistricting problems); *see* Quantifying Gerrymandering in NC, *supra* note 148, at 17–19 (reviewing validation tests).

The Duke team generated an ensemble of over 24,000 compliant redistricting plans that was submitted to the district court via VTD district assignment based around a precisely-chosen set of redistricting criteria.[157] Changing the relative importance of redistricting criteria may, in general, alter the conclusions of the initial ensemble. To test the robustness of the conclusions drawn from the primary ensemble of over 24,000 plans, many smaller ensembles were also generated and examined to verify that the relevant characteristics of the primary ensemble were insensitive to variations in redistricting criteria.[158] Such tests included, for example, reducing county splits in favor of slightly less compact districts.[159] In another test, an ensemble was generated that considered an alternative definition of compactness.[160]

In addition to validating that the results were insensitive to changes in the redistricting criteria, the Duke team also tested the robustness of the primary ensemble by showing, for example, that the results did not change when starting with different initial plans, nor did they change when over 100,000 samples were gathered. These two robustness tests help to demonstrate that the generated plans are well "mixed"; i.e., they are representative of the entire space, rather than merely some subset of it.[161]

---

157. *See* Quantifying Gerrymandering in NC, *supra* note 148, at 1.

158. *See id.* at 18–20.

159. *See id.*

160. *See id.*

161. Ensuring good "mixing" is a major challenge in generating ensembles. *See* Dana Randall, *Rapidly Mixing Markov Chains with Applications in Computer Science and Physics*, COMPUTING SCI. & ENGINEERING 30 (Mar./Apr. 2006). Several methodologies to aid mixing have been employed in generating ensembles of redistricting plans. Two such methods, parallel and simulated tempering, have been shown by a research team at Princeton University (now at Harvard) to be effective in generating well-mixed (though relatively small) ensembles using contiguity, population, and compactness criteria. *See* Fifield et al., *supra* note 156, at 17–30. Recent work by the Metric Geometry and Gerrymandering Group at Tufts University developed a new way to take a step in the random walk by using a split and merge technique on adjacent district pairs. Although the technical details are not yet published, this is a promising avenue forward in generating fast-mixing Markov Chains. Anthony Pizzimenti, Research at the Metric Geometry and Gerrymandering Group 6–9 (July 30, 2018) (unpublished manuscript), http://myweb.uiowa.edu/apizzimenti/files/documents/whitepaper.pdf (describing preliminary work on this approach). It is also possible to avoid the issue of mixing. A team at Carnegie Mellon University developed a localized theorem to test hypotheses that a given redistricting plan is atypical of the space of sampled redistricting plans. Instead of using an ensemble to understand what the partisan election results would have been in the absence of partisan gerrymandering, the idea is to test if a plan is atypical by comparing it with all local plans within a given random walk. Because only local plans are considered, one does not need to guarantee that random walk has mixed but can still draw conclusions from the ensemble. *See generally* Chikina et al., *supra* note 53.

C. Analyzing the North Carolina Ensemble

To analyze the partisan skew of the enacted plan relative to the ensemble, elections were simulated on each plan in the ensemble using partisan voting data from past elections. Although the use of partisan data neglects effects of incumbency and preferences for particular candidates,[162] these effects were substantially controlled by harmonizing results from diverse statewide races, including United States Senate, Governor, and Presidential races.[163] For each simulated election, the number of predicted Democratic and Republican officials were recorded along with their margins of victory. The team found that the simulated elections almost never resulted in as many as ten seats won by the Republicans, with the median number of Republicans elected varying gradually on the ensemble's median, from six to nine as the statewide Republican vote share shifted from forty-eight to fifty-six percent.[164]

Reflecting a district-by-district strategy of packing and cracking Democratic voters, the ensemble analysis revealed that the three most Democratic districts had significantly more Democratic votes than in any plan in the ensemble in the 2016 congressional election. On average, the three most Democratic districts in the ensemble held an average Democratic vote share of 61.32 percent, and the plan in the ensemble with the highest average Democratic vote share held 66.25 percent.[165] In contrast, the three most Democratic districts of the 2016 enacted plan held 67.93 percent of the vote share, suggesting these districts have been packed.[166] The next three most Democratic districts had fewer Democrats than in any plan of the ensemble.[167] On average, the next three most Democratic districts in the ensemble held an average Democratic vote share of 51.16 percent, and the plan in the ensemble with the lowest average Democratic vote share held

---

162. *Cf.* Vieth v. Jubelirer, 541 U.S. 267, 287 (2004) (noting that "[p]olitical affiliation . . . may shift from one election to the next; and even within a given election, not all voters follow the party line").

163. The defendants found it satisfactory to use a somewhat less diverse collection of partisan voting data from past elections in support of their declared goal of preserving a 10–3 seat advantage. *See* Common Cause v. Rucho, 318 F. Supp. 3d 777, 807 (2018) (quoting redistricting committee's criterion permitting use of election data from "statewide contests . . . not including the last two presidential contests"); *id.* at 806–07 (describing Lewis's acceptance of draft plan based on outcomes predicted by results of the 2014 United States Senate race between Thom Tillis and Kay Hagan).

164. *See* Quantifying Gerrymandering in NC, *supra* note 148, at 18–20.

165. *See id.* at 16.

166. *See id.*

167. *See* Joint Appendix at 383-84, Rucho v. Common Cause, No. 18-422 (S.Ct. filed Feb. 8, 2019).

45.72 percent.[168] In contrast, the next three most Democratic districts of the 2016 enacted plan held 42.96 percent of the vote share, suggesting that districts have been cracked of Democratic voters.[169] In fact, the fourth- through sixth-most Democratic districts would often elect Democrats in the ensemble, but were never predicted to do so in the enacted plan.[170]

The team's primary demonstrative exhibit at trial was the comparison of the 2016 enacted plan with a demonstration plan created by a bipartisan group of retired state judges[171] and with typical plans in the ensemble, as shown in Figure 1.[172] To construct this graph, the thirteen districts in each redistricting plan were ranked from lowest to highest by Democratic vote share. The ensemble of plans thereby gives rise to a collection of most Republican districts, a collection of second-most Republican districts, and so on, each of which has a distribution that can serve as a basis for comparison with the corresponding ranked districts in the enacted plan and in the retired judges' plan. The ordered comparisons reveal finer level details about the anomalous nature of the enacted plan. For example, the ensemble of plans reveals that the fourth- and fifth-most Democratic districts more often than not would elect Democratic representatives; the enacted map, in contrast, elects Republicans in these districts by over twelve percentage points. In addition to revealing anomalies in the winning party, the ensemble also reveals that election results may be solidified. In the sixth-most Democratic district, the Republican candidate in the enacted plan wins the election by a greater margin than in over 99.8% of plans in the ensemble. This abnormal and discriminatory arrangement of the electoral performance of North Carolina's congressional districts served to "consistently degrade[] [Democratic voters'] . . . influence on the political process as a whole."[173]

---

168. *See id.* at 351, 360. Percentages were not presented in the case but were calculated by Author Herschlag from the 24,000 ensemble maps that were entered into evidence. Data on file with Author Herschlag. *See also Rucho*, 318 F.Supp.3d at 872–83 (describing the 2012 and 2016 Democratic vote shares in Districts 9 and 13 as extremely low outliers in comparison with "the equivalent districts in the ensemble").

169. *See id.* Percentages were not presented in the case but were calculated by Author Herschlag from the 24,000 ensemble maps that were entered into evidence. Data on file with Author Herschlag.

170. *See id.*

171. *See* Press Release, Duke Sanford School of Public Policy, Nonpartisan Redistricting Panel Reveals Unofficial NC Congressional Voting Map (Aug. 29, 2016), https://sanford.duke.edu/articles/nonpartisan-redistricting-panel-reveals-unofficial-nc-congressional-voting-map.

172. *See also Rucho*, 318 F. Supp. 3d at 873 (reproducing a version of the graph).

173. *See* Davis v. Bandemer, 478 U.S. 109, 132 (1986).

D. Adjudicating the "Signature of Gerrymandering"

After trial, the three-judge panel found for the plaintiffs on all claims and enjoined the 2016 plan's use in the 2018 elections, with Judge Wynn authoring the majority opinion and Judge Osteen dissenting.[174] In adjudicating the plaintiffs' vote dilution theory, the majority found that the sharply contrasting curves successfully "demonstrated that the General Assembly 'cracked' and 'packed' Democratic voters."[175] First, the retired judges' plan closely aligns with the ensemble, providing validation that the ensemble's performance in elections is consistent with that of a plan drawn by skilled human drafters adhering to legitimate nonpartisan criteria.[176] Second, the enacted plan sharply deviates from the ensemble, reflecting the packing of Democratic voters into the three most Democratic districts and the cracking of Democratic voters in at least the three next-most Democratic districts.[177]

Synthesizing this evidence, the majority opinion referred to the *Rucho* plaintiffs' characterization of these district-by-district deviations between the enacted plan and the ensemble of plans as the "signature of gerrymandering,"[178] signifying discriminatory intent. The majority found the team's analysis to "provide strong evidence that the General Assembly intended to subordinate the interests of non-Republican voters and entrench the Republican Party in power."[179] The majority repeated this terminology in its analysis of discriminatory effects,[180] concluding from our analyses that "the 2016 Plan had a measurable tangible adverse impact on supporters of non-Republican candidates."[181]

---

174. *See* Common Cause v. Rucho, 279 F. Supp. 3d 587, 690 (M.D.N.C. 2018).
175. *See id.* at 643.
176. *See id.*
177. *See id.*
178. *Id.*
179. *Id.* at 644.
180. *See id.* at 657.
181. *Id.* at 666.

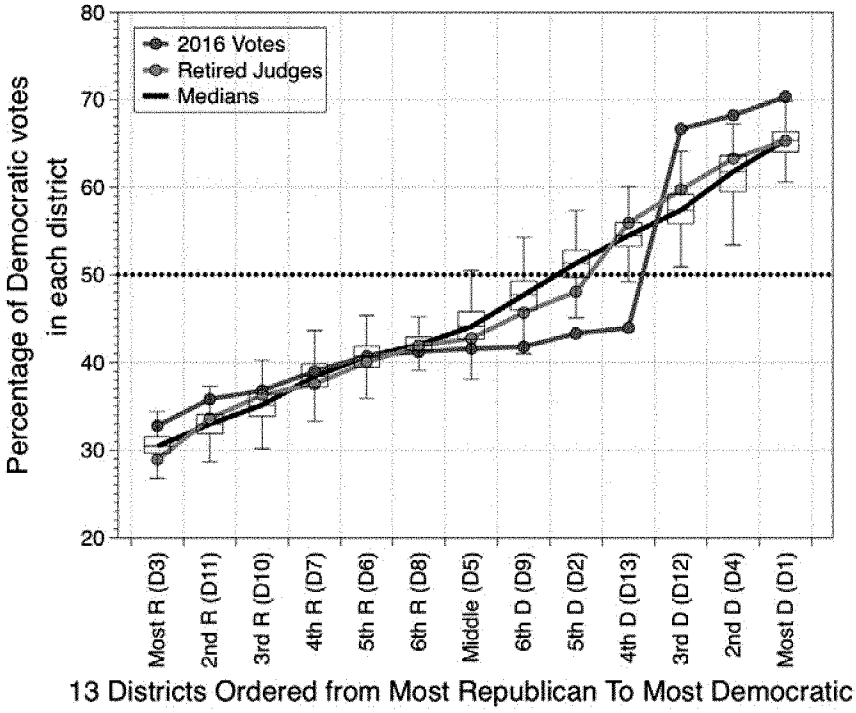

Figure 2: The "signature of gerrymandering." Collections of districts in the North Carolina ensemble are ranked in ascending order of Democratic vote share. The distribution of Democratic vote shares in each of the ranked districts of the ensemble is displayed with a standard box and whisker plot.[182] Overlaying the box plots are the ranked-votes curves of the enacted plan, the median, and a plan drawn by a bipartisan panel of retired judges.[183]

182. *See* Jonathan Mattingly, *Marginal Box-Plots: Summarizing What is Typical*, QUANTIFYING GERRYMANDERING (Feb. 11, 2018), https://sites.duke.edu/quantifyinggerrymandering/2018/02/11/marginal-box-plots-summarizing-what-is-typical.

183. *See* Press Release, *supra* note 171.

IV. THE ENSEMBLE ON REMAND

A. Gill's "Threshold Questions"

The *Rucho* defendants appealed the district court's decision to the Supreme Court,[184] which stayed the injunction[185] and ultimately vacated and remanded the decision for reconsideration in light of its standing decision in *Gill*.[186] *Rucho*'s remand proceedings therefore placed a special emphasis on addressing the issues adjudicated in *Gill*.[187]

In an opinion authored by Chief Justice Roberts,[188] the Court decided in *Gill* that the plaintiffs had failed to demonstrate standing to bring their claims that the Wisconsin legislature's 2011 redistricting violated the Fourteenth Amendment, without addressing the justiciability or merits of the claims themselves.[189] Describing partisan gerrymandering as a harm "to [the voter] as [an] individual[],"[190] the Court found that only four of the twelve voter-plaintiffs had complained of equal protection injuries specifically stemming from the packing or cracking of their own districts, and that all of the plaintiffs had incorrectly focused at trial on proving statewide harm. In light of these jurisdictional concerns, the Court vacated and remanded the Wisconsin district court's decision in *Gill*,[191] and summarily did the same with the North Carolina court's decision in *Rucho*.[192]

In a concurring opinion, Justice Elena Kagan sought to clarify the plaintiffs' burdens of proof in showing district-specific harms, both for the standing inquiry and for constitutional adjudication of their vote dilution claims on the merits.[193] Kagan offered that a plaintiff could meet the equal protection standing requirement by "show[ing] that her district has been packed or cracked," for example, by:

---

184. *See* Defs. Notice of Appeal, Common Cause v. Rucho, 279 F. Supp. 3d 587 (filed Jan. 11, 2018) (No. 1:16-cv-1026).

185. *See supra* note 44 and accompanying text.

186. *See supra* note 45.

187. *See generally* Common Cause v. Rucho, 318 F. Supp. 3d 777, 799 (M.D.N.C. 2018) (stating conclusion, on the first page of a 159-page opinion, that plaintiffs have standing "under the test set forth in *Gill*").

188. Gill v. Whitford, 138 S. Ct. 1916, 1922 (2018).

189. *Id.* at 1923.

190. *Id.* at 1930 (quoting Baker v. Carr, 369 U.S. 186, 206 (1962)) (alterations in original).

191. *Id.* at 1934.

192. *See supra* note 44 and accompanying text.

193. *See Gill*, 138 S. Ct. at 1934 (Kagan, J., concurring).

> produc[ing] an alternative map (or set of alternative maps)—comparably consistent with traditional districting principles—under which her vote would carry more weight. For example, a Democratic plaintiff living in a 75%-Democratic district could prove she was packed by presenting a different map, drawn without a focus on partisan advantage, that would place her in a 60%-Democratic district. Or conversely, a Democratic plaintiff residing in a 35%-Democratic district could prove she was cracked by offering an alternative, neutrally drawn map putting her in a 50–50 district. The precise numbers are of no import. The point is that the plaintiff can show, through drawing alternative district lines, that partisan-based packing or cracking diluted her vote.[194]

Gill thus left in place what Chief Justice Roberts characterized as the "two threshold questions [that] remain" from the Court's partisan gerrymandering jurisprudence: "what is necessary to show standing in a case of this sort, and whether those claims are justiciable."[195] In *Rucho*, the Duke team's ensemble analysis was instrumental in the district court's adjudication of each of these threshold questions on remand, as the following sections will show.

B. District-Specific Standing

On remand, the district court emphatically distinguished *Rucho* from *Gill* with respect to the district-specific standing requirement for equal protection claims based on the harm of vote dilution. The court unanimously concluded that, unlike the *Gill* plaintiffs who "failed to meaningfully pursue their allegations of individual harm,"[196] *Rucho*'s voter-plaintiffs resided in all of the challenged districts and "testified to, introduced evidence to support, and, in all but one case, ultimately proved the type of dilutionary injury the Supreme Court recognized in *Gill*."[197] What follows in this

---

194. *Id.* at 1936, 1939 (Kagan, J., concurring) (citations omitted). Nevertheless, Kagan concurred with the majority's refusal to adjudicate the statewide statistical evidence because "the plaintiffs tried this case as though it were about vote dilution alone" and "did not sufficiently advance a First Amendment associational theory." *Id.* at 1939.

195. *Id.* at 1929.

196. Common Cause v. Rucho, 318 F. Supp. 3d 777, 817 (M.D.N.C. 2018) (citing *Gill*, 138 S. Ct. at 1932).

197. *Id.* at 820; *see also id.* at 817–18 (discussing the theory and evidence presented by the Common Cause Plaintiffs); *id.* at 818–19 (discussing the theory and evidence presented by the League of Women Voters Plaintiffs); *id.* at 947 (Osteen, J., concurring in part and dissenting in part) (expressly stating concurrence with this conclusion).

section is a review of the evidence—including the Duke team's ensemble evidence—cited by the district court on remand in *Rucho* to support its conclusions regarding the district-specific standing requirement.

To assess the *Rucho* court's approach to district-specific standing, it is first necessary to recognize a latent ambiguity in the *Gill* Court's two descriptions of the dilutionary injury from partisan gerrymandering:

> To the extent the plaintiffs' alleged harm is the dilution of their votes, that injury is district specific. An individual voter in Wisconsin is placed in a single district. He votes for a single representative. *The boundaries of the district, and the composition of its voters, determine whether and to what extent a particular voter is packed or cracked.* This "disadvantage to [the voter] as [an] individual[]," therefore results from the boundaries of the particular district in which he resides.
>
> . . . .
>
> Here, the plaintiffs' partisan gerrymandering claims turn on allegations that their votes have been diluted. That harm arises from the particular composition of the voter's own district, which causes his vote—having been packed or cracked—to carry less weight than it would carry in another, hypothetical district.[198]

The ambiguity lies in the fact that the italicized sentence may be read as providing either an exhaustive or non-exhaustive list of the determinants of a vote dilution injury's existence and severity.

The exhaustive interpretation holds that "whether and to what extent a particular voter is packed or cracked" depends *only* on "[t]he boundaries of the [voter's assigned][199] district, and the composition of its voters."[200] The subsequent sentence, which causally attributes the dilution injury to "the boundaries of the particular district in which he resides,"[201] strongly supports this interpretation. It also implies that all voters of the same party and cracked or packed in the same district as the plaintiff suffer the same dilution injury, consistent with the Court's conclusion that the "burden arises

198. *Gill*, 138 S. Ct. at 1930–31 (internal citations omitted).

199. *See id.* at 1930 (providing the sentence: "An individual voter in Wisconsin is placed in a single district" as the referent for the definite noun phrase "the district").

200. *Id.*

201. *Id.*

through a voter's placement in a 'cracked' or 'packed' district."[202] Further supporting this reading is the Court's finding that four of the *Gill* plaintiffs had adequately pled vote dilution claims by virtue of alleging that they "live in districts where Democrats like them have been packed or cracked."[203]

The non-exhaustive reading holds that "whether and to what extent a particular voter is packed or cracked" may also depend on the specific location of the voter's residence. Support for this view comes from the fact that the range of hypothetical districts against which it is reasonable to compare the voter's district in a vote-dilution theory[204] is sensitive to the voter's location within the district. For example, natural packing is more likely to confound remedial redistricting efforts if the voter resides in the core of an urban district rather than its suburban periphery.[205] Compared with the exhaustive interpretation's district-based approach, a location-specific theory of vote dilution supports Article III standing at least as well *a fortiori*, insofar as it is "particularized"[206] to the plaintiff's household and not "dispensed in gross."[207]

The "signature of gerrymandering" addresses only the first, exhaustive interpretation of the *Gill* Court's description of vote dilution. It is well-suited for identifying enacted districts in which packing and cracking has occurred, but it does not immediately identify the specific locations of voters whose influence has been diluted relative to the influence they would have had under a more typical redistricting.[208] For example, in North Carolina, the ensemble revealed that it was typical to have three districts with a fifty-five to sixty percent Republican vote share in the 2016 congressional general election; however, the 2016 enacted plan had six districts in this range, and three of these six districts exhibited cracking of Democrats relative to their

---

202. *Id.* at 1931.

203. *See id.* at 1931 (finding that Mary Lynne Donohue, Wendy Sue Johnson, Janet Mitchell, and Jerome Wallace "pleaded a particularized burden" from vote dilution); *id.* at 1934 (remanding to allow plaintiffs "to prove concrete and particularized injuries" at trial).

204. The injury-in-fact requirement for standing, which excludes "conjectural or hypothetical" harms, *see* Lujan v. Defs. of Wildlife, 504 U.S. 555, 560 (1992), logically necessitates some limitations on the range of hypothetical districts that can be used to support any particular vote-dilution theory.

205. *See Gill*, 138 S. Ct. at 1936; *see also Unintentional Gerrymandering*, *supra* note 53, at 266 (describing constraints on remedial redistricting imposed by political geography).

206. *See Lujan*, 504 U.S. at 560 (stating standing requirement of "concrete and particularized" injury).

207. *See Gill*, 138 S. Ct. at 1934 (quoting DaimlerChrysler Corp. v. Cuno, 547 U.S. 332, 353 (2006)) ("Standing is not dispensed in gross . . . .").

208. *See* John Mattingly, *Towards a Localized Analysis*, QUANTIFYING GERRYMANDERING (July 12, 2018), https://sites.duke.edu/quantifyinggerrymandering/2018/07/12/towards-a-localized-analysis.

counterparts in the ensemble.[209] Accordingly, the *Rucho* court credited the "signature of gerrymandering," *inter alia*, with supporting most of the plaintiffs' district-specific claims by showing that all but one of the enacted districts "reflect[] a successful effort by the General Assembly" either "to concentrate, or pack, voters who were unlikely to support a Republican candidate, and thereby dilute such voters' votes"[210] or "to crack likely Democratic voters and thereby dilute their votes by submerging them in a safe Republican district."[211]

Serving as an expert witness for the League of Women Voters, Jowei Chen filed a supplemental declaration during the remand proceedings that addressed the second, non-exhaustive reading of the *Gill* Court's characterization of vote dilution.[212] Specifically, Chen prepared a table comparing the average Republican vote shares—based on the election data used by Hofeller—in the district containing each plaintiff's precinct under the enacted plan and in the district that would contain the plaintiff's precinct in a demonstration plan.[213] Chen also performed similar comparisons with each of 2,000 other simulated districting plans.[214] In the district-by-district standing analysis, the *Rucho* court credited these comparisons with showing that the plaintiffs' votes "would have carried greater weight in numerous other 'hypothetical district[s].'"[215]

Without expressly acknowledging the ambiguity in the *Gill* Court's characterization of voting dilution, the *Rucho* court's adjudication of the district-specific standing requirement thereby addressed both interpretations of the opinion. We believe the more natural reading of the opinion, especially in light of the Court's adjudication of the four district-specific claims,[216] is that voters throughout a cracked or packed district have equal standing to challenge the district's boundaries and composition—and that the specific location of a voter-plaintiff's residence comes into play only later as it influences the range of hypothetical districts capable of remedying the plaintiff's injury. Regardless of how the ambiguity is ultimately resolved

---

209. *See* Quantifying Gerrymandering in NC, *supra* note 148, at 3–4.

210. *See* Common Cause v. Rucho, 318 F. Supp. 3d 777, 821 (M.D.N.C. 2018).

211. *See id.* at 823.

212. *See* Suppl. Decl. of Jowei Chen, App. 2 to League of Women Voters Pls.' Br., League of Women Voters v. Rucho (filed July 11, 2018) (No. 1:16-CV-1164-WO-JEP), *aff'd sub nom.* Common Cause v. Rucho, 318 F. Supp. 3d 777 (2018), https://campaignlegal.org/document/rucho-v-league-women-voters-north-carolina-us-district-court-middle-district-north-1.

213. *See id.* at 6.

214. *See Rucho*, 318 F. Supp. 3d at 821 (citing Third Chen Decl. 4, 6–8, 11).

215. *See id.* (quoting Gill v. Whitford, 138 S. Ct. 1916, 1931 (2018)).

216. *See supra* text accompanying note 203.

on appeal, however, the combination of the Duke team's district-based analysis of packing and cracking—and Jowei Chen's localized comparisons with hypothetical districts—will support the plaintiffs' standing to bring their vote dilution claims.

C. Justiciability

On remand, the *Rucho* defendants contested the justiciability of ensemble analysis as a basis for inferring discriminatory intent and effect, characterizing this approach as "'a smorgasbord of alleged 'social science' theories' that lack any constitutional basis."[217] The district court responded with a ringing defense of the utility of statistical evidence in adjudication to vindicate core constitutional values:

> Legislative Defendants are correct that none of these empirical analyses appear in the Constitution. But Plaintiffs need not show that a particular empirical analysis or statistical measure appears in the Constitution to establish that a judicially manageable standard exists to resolve their constitutional claims. Rather, Plaintiffs must identify cognizable constitutional standards to govern their claims, and provide credible evidence that Defendants have violated those standards. And contrary to Legislative Defendants' assertions, Plaintiffs do not seek to constitutionalize any of the empirical analyses they have put forward to support their claims, nor does this Court do so. Instead, Plaintiffs argue that these analyses provide evidence that the 2016 Plan violates a number of well-established constitutional standards—that the government act impartially, not infringe the right to vote, not burden individuals based on the exercise of their rights to political speech and association, and not allow state legislatures to dictate electoral outcomes or interpose themselves between the voters and their representatives in Congress.[218]

The court proceeded to recount prominent examples of constitutional claims that had crucially depended on statistical and social science evidence.[219]

As one of us argued as an *amicus* in *Gill*, causal inference based on statistical evidence is particularly well-suited to the adjudication of

---

217. *Rucho*, 318 F. Supp. 3d at 852 (quoting Leg. Defs. Post-Trial Br.).

218. *Id.* at 853 (citations omitted).

219. *See id.* at 853–54 (citing, *inter alia*, Yick Wo v. Hopkins, 118 U.S. 356 (1886); Brown v. Bd. of Educ., 347 U.S. 483 (1954)).

constitutional challenges to computer-assisted redistricting.[220] Recognizing this, the *Rucho* court acknowledged that "the judiciary . . . has an obligation to keep pace with technological and methodological advances so it can effectively fulfill its constitutional role to police ever more sophisticated modes of discrimination."[221] The court characterized our ensemble approach and other empirical methods introduced at trial as the kinds of "new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties," as foretold by Justice Kennedy in *Vieth*.[222]

V. CONCLUSIONS AND FUTURE WORK

The Supreme Court's partisan gerrymandering jurisprudence in the thirty-five years since *Bandemer* has left open the possibility of a justiciable quantitative standard,[223] but it has cabined this possibility with a growing recognition of alternative explanations for partisan disparities in electoral performance[224] and, in *Gill*, with the characterization of vote dilution as a district-specific injury.[225] Previous quantitative measures of gerrymandering have generally failed to address these jurisprudential concerns insofar as they have been predicated on *a priori* assumptions about the relationship between the seats won by a political party and the votes cast for that political party.[226]

Although automated redistricting tools have been around since the 1960s,[227] generating an ensemble of comparison plans from a known distribution is a relatively new field that contains a plethora of open and interesting scientific questions. Such questions involve how best to sample from the space,[228] and even on the choice of distribution and its effect on different classes of conclusions. For example, the Duke team used a weighted distribution that favored more compact plans.[229] Other research teams, such as those at Carnegie Mellon University[230] and Tufts

220. *See Gill* Amicus Brief, *supra* note 19, at 6.
221. *Rucho*, 318 F. Supp. 3d at 856.
222. *See id.* at 856 (citing Vieth v. Jubelirer, 541 U.S. 267, 312–13 (2004)).
223. *See supra* text accompanying notes 58–61.
224. *See supra* text accompanying notes 79–81.
225. *See supra* text accompanying note 190.
226. *See supra* text accompanying notes 101–107.
227. *See* Quantifying Gerrymandering in NC, *supra* note 148, at 7.
228. *See* Randall, *supra* note 161 (discussing open problems relating to ensuring that the random walk used to gather the sample "mixes" well).
229. *See supra* text accompanying note 155.
230. *See* Chikina et al., *supra* note 53.

University,[231] prefer to use uniform distributions that give equal weight to all plans below a specified compactness threshold.[232] Ultimately, it is still unknown whether there are circumstances in which the choice of distribution will affect the conclusions drawn from an ensemble; however, preliminary results suggest the conclusions are remarkably robust. Despite these open scientific questions, we can already test for robustness and sensitivity of ensembles on a case-by-case basis to test the validity and robustness of the conclusions drawn.

The proceedings in *Rucho* thus far have demonstrated the potential utility of ensemble analysis in federal constitutional adjudication of partisan gerrymandering claims, paralleling last year's election-changing litigation in Pennsylvania. As the case returns to the Supreme Court on appeal, we are optimistic that the fundamental shift in the quantitative analysis of gerrymandering from partisan asymmetry measures to the ensemble approach will prove to be an enduring and productive one.

The *Rucho* appeal presents the Court with its first opportunity to review the ensemble approach to identifying partisan gerrymandering and quantifying its effects. Ensemble approaches make no *a priori* assumptions about votes-seats relationships, and, in particular, do not assume proportionality. Instead, ensemble approaches take stipulated legal redistricting criteria as the starting point for ensemble generation and analysis, and reveal atypical electoral performance in individual districts as well as in the statewide outcome. Because political geography that results in "natural" partisan disparities affects ensemble maps in the same way that it affects the challenged map, political geography can be eliminated as an alternative explanation for a challenged map's outlier status. By addressing these jurisprudential concerns and by identifying and quantifying harms at a district-specific level through the "signature of gerrymandering," ensemble analysis holds considerable promise for understanding and adjudicating gerrymandering disputes.

---

231. *See generally* Metric Geometry and Gerrymandering Grp., *supra* note 53.
232. *See id.* at 21.